Health-related U-shaped time preference and consumption profile

研究ノート

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Kei Hosoya*

▶ Abstract

In this paper, we summarize and discuss the arguments behind using the time preference function, which assumes that an individual's time preference rate is endogenously determined by their stock of health deficits, and we introduce a specific function that can reproduce the characteristics revealed in this discussion. There can be periods of weakening and strengthening of the present preference during an individual lifetime. The corresponding typical time preference curve shows a U-shape throughout one's lifetime. In particular, the decreasing phase of the discount rate in young adults is caused by hyperbolic discounting. On the other hand, from the middle to elderly ages, the combined effects of aging and senescence lead to an increase in the rate of time preference. The characteristic time preference function so derived is implemented in a dynamic model that takes into account the health-damage component, and its impact on the lifetime consumption profile is examined under different health-related scenarios. Although each scenario is realistically possible, there are notable differences in the projected consumption profiles.

▶ JEL classification numbers:

D91; E21; I12; O40

▶ Keywords:

Endogenous time preference; Health deficit; Hyperbolic discounting; Increasing marginal impatience; Lifetime consumption

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1. Introduction

In this paper, we investigate the properties of the rate of time preference (time discount rate), an important deep parameter of economic models, by allowing it to vary over time. Exponential discounting, common in standard optimal growth models, is typically implemented in the model, and thus the rate of time preference is constant over time. This simple assumption, while superior in terms of mathematical tractability, raises questions based on recent findings in behavioral and experimental economics.

Even brief observations of the economic behavior of ourselves, our neighbors, and people around the world suggest that the assumption of a constant rate of time preference across a lifetime, representing the degree to which we discount the future, is rather restrictive. Nevertheless, it is not appropriate to assume that it exhibits completely random trends, and it seems natural to assume that the magnitude of the rate of time preference reflects the characteristic tendencies of each age group in accordance with their general circumstances.

What, then, is the characteristic tendency? If it is neither constant nor random, then we can distinguish between trends with persistently decreasing values and trends with persistently increasing values over a certain period of time. In what follows, we examine this issue from several perspectives, including the underlying mechanism, and derive an appropriate time preference function. Using this, we attempt to connect it to economic growth, a typical problem in which time preference plays a major role, and to confirm its empirical implications.

All of this indicates that the rate of time preference is now positioned as an endogenous variable in the model. In other words, our ideas can be reformulated within the framework of endogenous time preference (see Uzawa, 1968; Epstein and Hynes, 1983; Obstfeld, 1990; Das, 2003; Meng, 2006). The most prevalent case is the well-known mechanism by which the rate of time preference is endogenously determined depending on standard economic variables, such as the level of consumption (e.g., Nishimura and Shimomura, 2002). In this regard, we follow in the steps of Strulik (2021) and consider the rate of time preference as depending mainly on gradually accumulating individual health deficits. Biological senescence, as distinguished from aging, is the true nature of health deficits. There are two major reasons, unique to this paper, for focusing on health deficits. One is what we refer to as the age effect, in which preferences are formed in different directions depending on the age group to which an individual belongs. The other, which we call the health cognition effect, may also lead to complicated preference formation. These will be discussed in detail in the next section.

2. Endogenous time preference related to aging and health

Our focus is on the formation of preferences for individual agents with an emphasis on health factors, and we summarize them into a unique decision pattern, albeit via a complex process in terms of content. In this section, we examine the process in detail and consider the specification of a particular time preference function.

One important mechanism is the age-group effect, which focuses on the age group to which a given individual belongs and is generally related to *aging*. Young adults may tend to prioritize current pleasures in their daily lives because they are generally far from the expected time of death, which is often *not* immediate concern for them. If we assume that this tendency is dominant, then the rate of time preference for young adults will be high. Under this assumption, future preference may be more important for middle-aged adults. Although their health status may be declining somewhat compared to younger adults, they need to live with the prospect of a much longer retirement in modern times. This background factor seems to be directly and closely related to the fact that they as a generation have a higher propensity to save. It is also interesting to examine the formation of preferences among the elderly in terms of age-group effects. As a generation living with a clear sense of mortality, it is unlikely that they have a strong preference for the future unless they have a special bequest motive, in which case, they, in a sense, live in the present as carefully as young adults (i.e., they have a higher present preference).

In summary, a U-shaped curve is expected to be shown for the time preference rates of representative agents over their lifetimes based on the age-group effect. In particular, the well-known concept of hyperbolic discounting can explain the decreasing trend of rates in younger ages (see e.g., Laibson, 1997; Strulik and Trimborn, 2018).

Now assume that the discount function, $R(t)$, is a differentiable, strictly decreasing function satisfying $R(0)=1$. As the most basic example, $R(t)$ can be formulated as an exponential function

$$R(t)=\exp(-\rho t), \tag{1}$$

where $\rho>0$ is a constant rate of time preference (discount rate). Writing this in a general manner in an unrestricted form, as in (1), we derive the following relation with $\rho(t)$ as an instantaneous rate of time preference:

$$R(t)=\exp\left(-\int_0^t \rho(s)\,\mathrm{d}s\right)$$

$$\Longleftrightarrow \quad \rho(t) = -\frac{R'(t)}{R(t)}. \tag{2}$$

When $R(t)$ is an exponential function, $\rho(t)$ is constant.[1] It has been pointed out that exponential discounting under a constant rate of time preference is not appropriate for individual preferences across intertemporal decision-making. In this regard, there is a relatively large number of studies that argue for the empirical validity of decreasing marginal impatience (DMI), in which the rate of time preference decreases over time, and in the context of behavioral economics, hyperbolic discounting appears to be consistent with this decreasing nature (Anchugina et al., 2017). To broadly characterize hyperbolic discounting, let us consider the ***proportional*** hyperbolic formulation.

Assuming that $R(t)$ is the proportional hyperbolic form, we specify

$$R(t)=\frac{1}{1+mt}, \tag{3}$$

where $m>0$ is a hyperbolic discount rate. Equations (2) and (3) lead to

$$\rho(t) = -\frac{R'(t)}{R(t)} = -\frac{\dfrac{-m}{(1+mt)^2}}{\dfrac{1}{1+mt}} = \frac{m}{1+mt}. \tag{4}$$

From (4), we can see that $\rho(t)$ decreases over time, which clearly shows the charac-

1 From (1) and (2), $-R'(t)/R(t)=-(-\rho)\exp(-\rho t)/\exp(-\rho t)=\rho$ is obtained.

teristics of hyperbolic discounting. As pointed out above, the early phase of a lifetime focusing on the age-group effect can be considered as corresponding to the depiction in which hyperbolic discounting is one of the dominant mechanisms. This image is shown in Figure A1 of Appendix A.

The vertical axis represents the size of the rate of time preference, and the situation in this figure is expected to be true for the early stages of a person's life up to their 50s. In the later stages of life, however, we expect that an increase in present preference specific to the elderly will be added to the situation shown in Appendix A. What is clear from this discussion is that it is unlikely that the rate of time preference will remain constant throughout one's lifetime.

Another important mechanism is the health cognition effect, which is distinct from aging and related to how the ***perception*** of personal health status is linked to health deficits. This effect may be related to biological senescence (frailty) in a general sense, although it may also be affected by the experience of a severe illness.[2] A higher health cognition indicates that an individual is more likely to be able to live thoughtfully both in the present and in the future. In other words, when this cognition effect is high, present preference is assumed to be relatively weak and the rate of time preference is assumed to be low. However, while it is conceivable that individuals with low health cognition may carefully and deliberately lead long lives, in general, the more they senesce and the lower their health cognitions, the more likely they are to choose an ephemeral way of life, coupled with the effect of general aging.

Taking all of this into account, we can assume that this effect is not pronounced and is not dominant in young adults. Hence, the effect on their time preference is negligible. In a relative sense, although health awareness increases when one reaches middle age, the cognition of health is generally higher, and the effect of increasing the rate of time preference may be small. However, unlike young adults, the effect cannot be considered negligible. For older ages, health cognition usually declines with the accumulation of health deficits (senescence), although there may be exceptional cases. This implies a gradual increase in the rate of time preference (corresponding to increasing marginal impatience). As in the case of the age-group effect, let us predict the evolution of the rate of time preference. In this case, the rate of time preference is

2 If people are living healthy and active lives, then it is possible that they may have high health cognitions, even as they become elderly.

expected to bottom out during middle age and to increase monotonically as people continue to age. This is shown in Figure B1 of Appendix B.

Based on the previous discussion, one possibility is to assume that an individual's subjective time preference rate is expressed by a combination of the age-group effect and the health cognition effect. An example of this is shown in Figure C1 of Appendix C. This indicates that the rate of time preference is initially relatively high in young adults but then declines toward middle age, and after that period, it continues to increase in the elderly. As a result, it is expressed as a U-shaped time preference function.

What do previous studies suggest regarding our inference? Structurally, we expect that decreasing marginal impatience dominates the determination of time preference in early life stages, while increasing marginal impatience strengthens in later life stages. Based on field experiments in Denmark, Harrison et al. (2002) confirmed that a U-shaped relationship can be found for the rate of time preference with middle age at its lowest point. Additionally, there was a study by Hirata (2008) that observed a relationship similar to this in Japan. The possibility that the rate of time preference will vary with different age groups and that it will exhibit a U-shape is theoretically and empirically meaningful for verification. If this is a reliable finding, then it would have important implications for policy planning in many fields.

A concrete functional form

The following function proposed by Strulik (2021) is an example of a time preference function indicating a U-shape:

$$\rho = \bar{\rho} + \underbrace{\eta_1 \exp(-\eta_2 \boldsymbol{D})}_{\text{DMI}} + \underbrace{\eta_3 \exp(\eta_4 \boldsymbol{D})}_{\text{IMI}}, \tag{5}$$

where $\bar{\rho}$ is the baseline value and $\boldsymbol{D}$ is the index of health deficits stocks. In (5), $\eta_1 \exp(-\eta_2 \boldsymbol{D})$ is a term that captures the decreasing trend of the rate of time preference with increasing $\boldsymbol{D}$, and it represents the hyperbolic discounting caused by the age-group effect. For the motion of ρ, it shows decreasing marginal impatience with respect to $\boldsymbol{D}$.[3] In contrast, $\eta_3 \exp(\eta_4 \boldsymbol{D})$ is an exponentially increasing term that describes increasing marginal impatience. Our stance in this paper is that the degree of

3 This is a simplification and reflects the assumption that aging and the accumulation of $\boldsymbol{D}$ are roughly parallel.

Table 1. Health deficits at each age point.

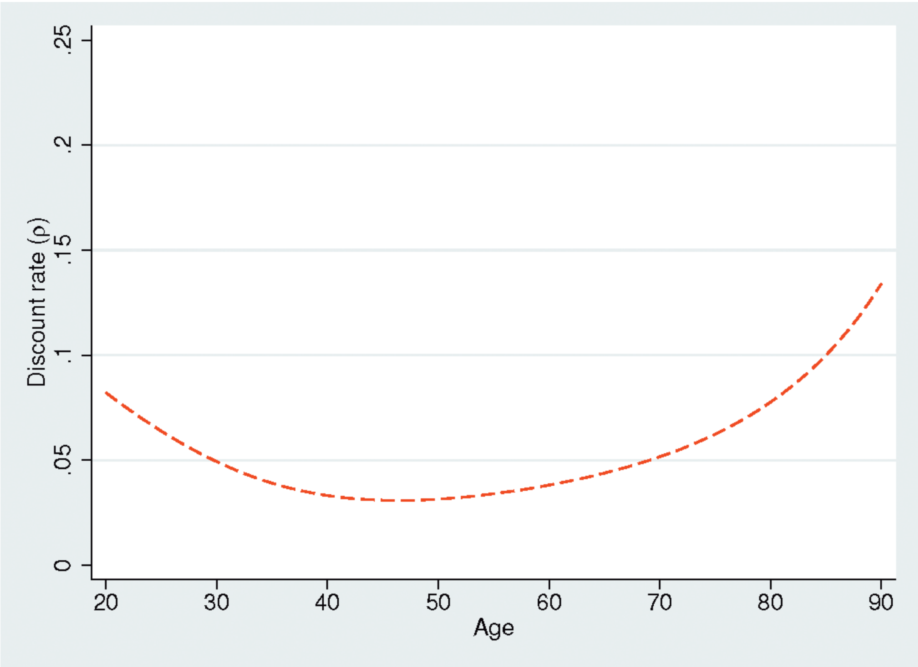
Age	20	30	40	50	60	70	80	90
D	0.0270	0.0307	0.0362	0.0448	0.0577	0.0775	0.1075	0.1534

these effects is related to the age-group effect and the health cognition effect as background factors.

We would like to examine two patterns here that will pertain to the analysis in the following section: (i) a case in which the minimum occurs at middle age and both young adults and the elderly are at the same level, and (ii) a case in which the minimum is also set at middle age but the rate for the elderly is higher. As a preliminary preparation, it is necessary to determine the evolution of health damages D at each age point. Specifically, the change follows $\dot{D}=\nu(D-d)$ with ν and d as constants.[4] Values at each age, D, are shown in Table 1.

Case (i) is depicted in Figure 1. By devising a combination of parameters included in (5), a time preference curve can be generated in which the rates for younger and older age groups do not differ significantly. Case (ii) is expected to be more standard.

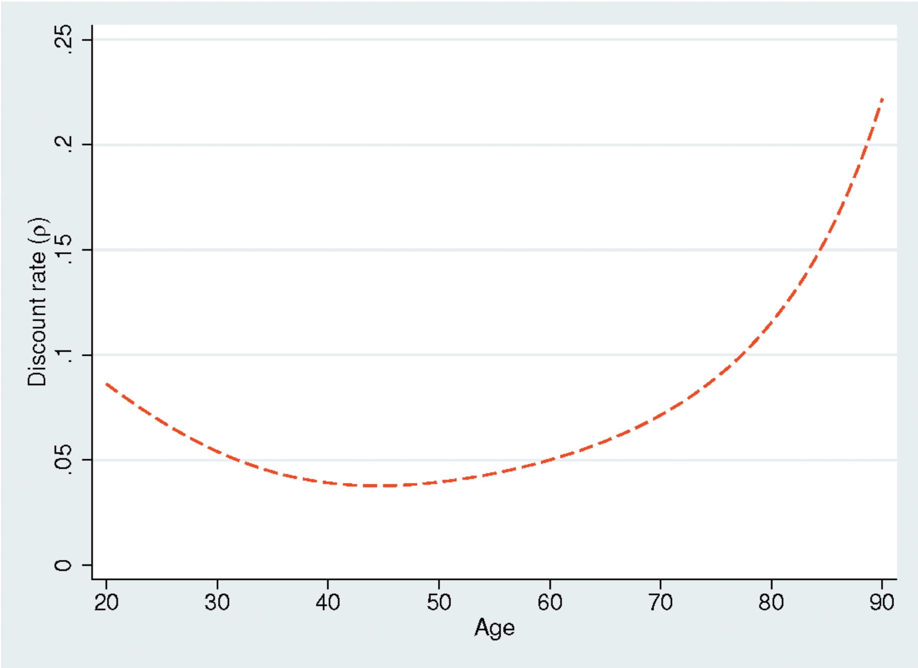
Figure 1. U-shaped time preference (i).



Note: $\bar{\rho}=-0.03$, $\eta_1=30$, $\eta_2=229.65$, $\eta_3=0.04$, and $\eta_4=9.2$.

4 For more details, see Mitnitski et al. (2002), Dalgaard and Strulik (2014), Strulik (2021), and Hosoya (2023). We set $\nu=0.043$, $d=0.02$, and $D_0=0.027$ (the initial value of health deficits).

Figure 2. U-shaped time preference (ii).



Note: $\bar{\rho}=-0.03$, $\eta_1=30$, $\eta_2=229.65$, $\eta_3=0.04$, and $\eta_4=12$.

Using case (i) as a benchmark and adjusting the value of η_4 in particular, we can create the curve shown in Figure 2, which is consistent with our intention.

Thus, using the specification in (5), it is possible to represent the assumed scenario with some flexibility by devising combinations of $\bar{\rho}$ and η_1–η_4. In the next section we examine differences in lifetime consumption profiles under this framework.

3. Time preference and lifetime consumption profiles

While the variation of time preference rates over a lifetime is interesting in itself, it is important to see how a time preference function with these characteristics performs when it is implemented in a dynamic general equilibrium model. Strulik (2021) solved a growth model consistent with the setting used in this paper and showed that the Euler equation for consumption (i.e., the Keynes-Ramsey rule), which specifies the growth rate of consumption under health deficit considerations, can be derived as

$$\frac{\dot{c}}{c}=\frac{r-\rho(D)}{\theta}, \tag{6}$$

where r is the rate of return on physical capital (the interest rate), and θ is the inverse of the intertemporal elasticity of substitution for consumption. Equation (6) differs from the standard optimal growth model in that the rate of time preference, ρ, is endogenously determined depending on D (i.e., $\rho(D)$). It is interesting to note that (6) holds with or without the implementation of individual health investment.[5]

From (6), with $c(0)$ as the initial value of consumption, the individual consumption level, $c(t)$, can be written as

$$c(t)=c(0)\cdot\exp\left[\frac{1}{\theta}\{r-\rho(D)\}t\right]. \tag{7}$$

We now consider three scenarios. As a comparison, we set up an intermediate case between the cases shown in Figure 1 and Figure 2 above. Specifically, one in which the rate of time preference for the elderly is higher than that of young adults, but the difference is only about twice as large. This case serves as the benchmark and is obtained under $\bar{\rho}=-0.03$, $\eta_1=30$, $\eta_2=229.65$, $\eta_3=0.04$, $\eta_4=10.5$, $c(0)=4{,}000{,}000$, $r=0.08$, and $\theta=2.5$. Throughout this paper, "consumption data" refers to data from the ***Family Income and Expenditure Survey*** by the Statistics Bureau of Japan. The interest rate was set based on the results of a comprehensive study by Jordà et al. (2019), while also referring to other literature. The inverse of the intertemporal elasticity of substitution was then set within an appropriate range, based on Barro et al. (1995), Stokey and Rebelo (1995), Ortigueira and Santos (1997), and Raurich (2003), among others.[6] The level of consumption at each age is expressed as a ratio relative to that of the age at which consumption peaks, normalized to 1 (so as not to be affected by differences in the initial value, $c(0)$).

A case with an exceptionally large value for young middle ages: Scenario 1

This applies to cases with a fairly long life expectancy (i.e., the age-group effect is more pronounced) and, in general terms, to cases where people tend to choose ephemeral consumption patterns due to strong optimism or strong pessimism. The es-

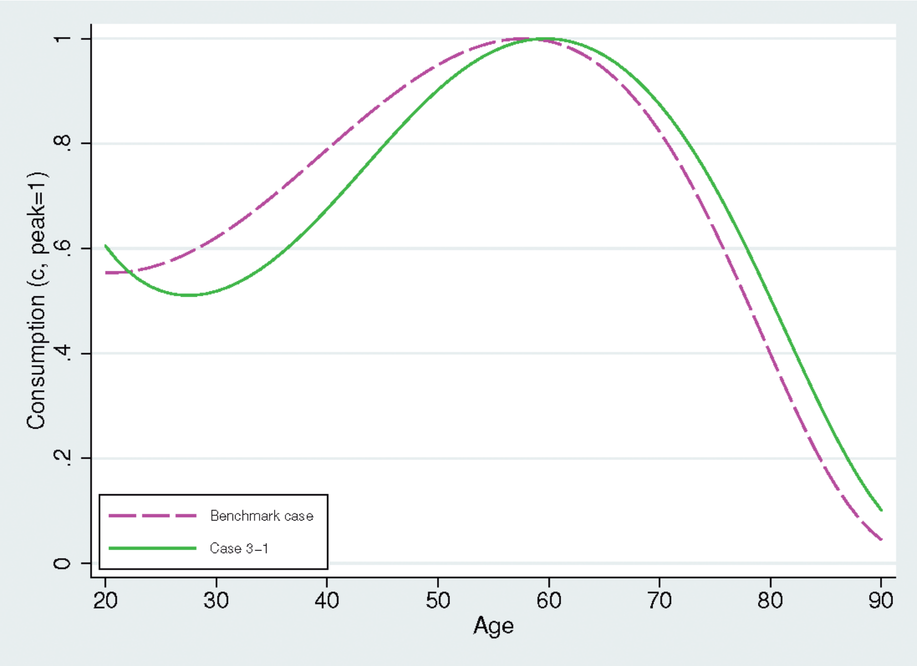
5 Hosoya (2023) analyzes the impact of the recent COVID-19 pandemic on lifetime consumption by extending Strulik's (2021) individual health investment model to include public health expenditures. With this extension, (6) is modified so that

$$\frac{\dot{c}}{c}=\frac{(1-\tau)r-\tau m-\rho(D)}{\theta},$$

where τ is the income tax rate and m is the mortality rate.

6 See Appendix D for cases in which attempts have been made to change the values of economically important parameters.

Figure 3. Scenario 1.



Note: Case 3-1 is depicted under $\bar{\rho} = -0.02$, $\eta_1 = 24.92$, $\eta_2 = 185$, $\eta_3 = 0.04$, $\eta_4 = 9.2$, $c(0) =$ 4,000,000, $r = 0.08$, and $\theta = 2.5$.

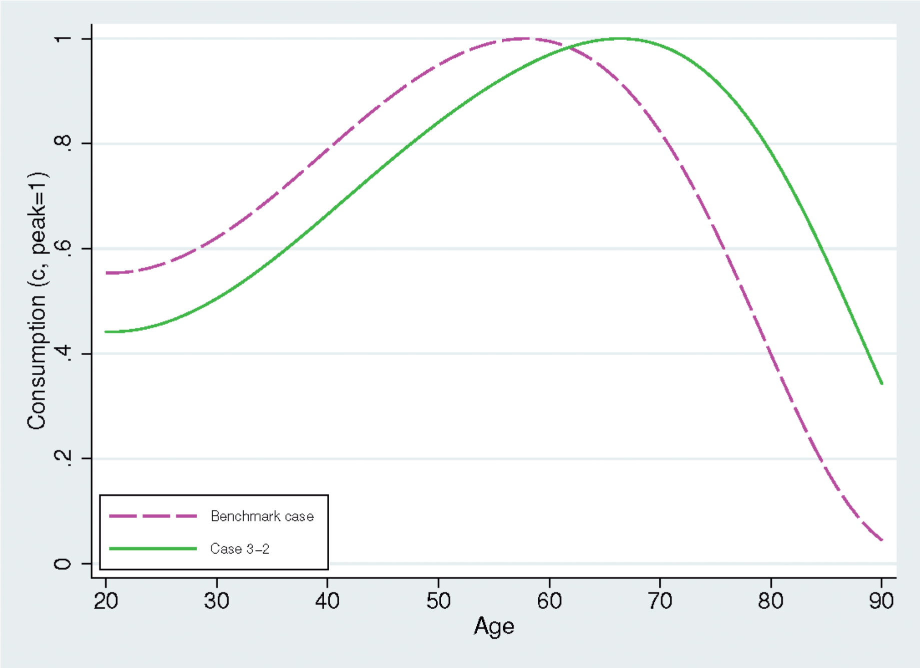
timated result for the lifetime consumption profile is shown in Figure 3.

One point to note when comparing the results is that the two cases differ in the age at which consumption peaks and, naturally, in the level of consumption. In the benchmark case, it is about 7,220,000 at age 58, and in Case 3-1, it is about 6,610,000 at age 60. Nevertheless, as shown in Figure 3, Case 3-1 reflects a strong present preference with consumption rising only at the beginning, but at a significant overall reduction in consumption from young adults to middle age. This is an important observation. Within Case 3-1, the rate of time preference in the elderly period is relatively low, so that consumption in this period is similar to the pattern in the benchmark case. The choice of extreme consumption patterns due to unique preference formation is still not highly desirable.

A case of low and stable values over the lifetime: Scenario 2

This case corresponds to a situation in which the health cognition effect is high over a long period and, therefore, physical and mental health is maintained (Case 3-2). Given that our setting is a representative agent model, it seems to match a situation in

Figure 4. Scenario 2.



Note: Case 3-2 is depicted under $\bar{\rho}=-0.027$, $\eta_1=62$, $\eta_2=253.51$, $\eta_3=0.038$, $\eta_4=7.28$, $c(0)=4,000,000$, $r=0.08$, and $\theta=2.5$.

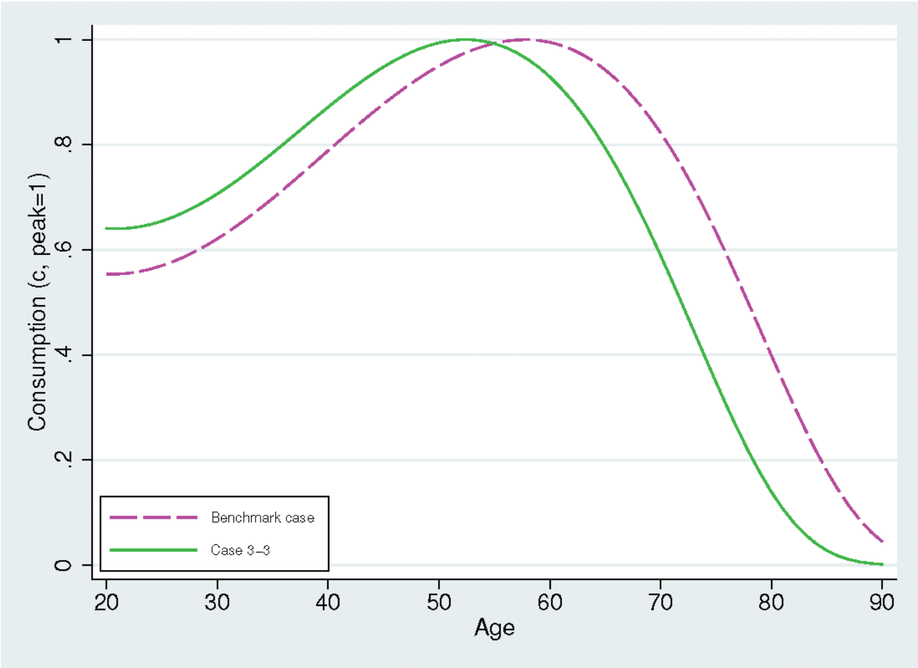
which a country as a whole has a high level of health and a good social security system. Figure 4 shows the result, and we can clearly confirm the difference relative to the benchmark case.

Since this can be considered a case of strong future preference at most points in life, it can be inferred that this is indicative of fairly affluent consumption in the elderly periods. This favorable economic outcome is the *product* of people's patient preferences.

A case with an exceptionally large value for the elderly: Scenario 3

It is natural that the elderly, who are aware of the end of their lives, have strong present preferences, but what are the cases that correspond to exceptionally strong situations? For situations concerning health issues and the elderly, it is undeniable that their rate of time preference may increase significantly when the content of the health care system for them becomes more severe due to tight social security finances. In addition to the fact that a higher rate of time preference can naturally be explained by the age-group effect, the more fragile elderly health care may result in a lower cogni-

Figure 5. Scenario 3.



Note: Case 3–3 is depicted under $\bar{\rho}=-0.03$, $\eta_1=30$, $\eta_2=229.65$, $\eta_3=0.0388$, $\eta_4=13.94$, $c(0)=4{,}000{,}000$, $r=0.08$, and $\theta=2.5$.

tion of health, which may further support the stronger present preference. Figure 5 shows the corresponding estimation result (Case 3–3).

The time preference curve in this scenario is entirely U–shaped, about 0.08 at the highest value for young adults and about 0.04 for the middle age range, with only the elderly being prominently higher (above 0.1 at age 75 and above 0.25 at age 88). The characteristics of this curve confirm that the impact is long–lasting, well into the 50s and beyond, which is longer than expected. If the sustainability of the social security system, including the medical care system, is obstructed, then serious effects can be expected for many aspects of it, but the impact on consumer life, as revealed in this scenario, cannot be overlooked.

4. Concluding remarks

In this paper, we studied the important topic of endogenous time preference, where the rate of time preference is determined by an indicator of health, and examined its impact on lifetime consumption profiles. The index of health was expressed as health deficits, which may either decrease or increase the value of the rate of time preference as they accumulate. The former was characterized by hyperbolic discounting and exhibited decreasing marginal impatience as a functional property, whereas the latter exhibited increasing marginal impatience because it was characterized by strong present preference, representing more impatient decision making. This allowed us to obtain a U-shaped time preference curve throughout all ages.

We have given a unique interpretation of the time preference function based on the background of health and its related factors. Namely, the age-group effect explains hyperbolic discounting in younger adults and, conversely, the increase in present preference for older adults. Nonetheless, since aging is different from senescence, it is a somewhat crude assumption to assume that all the elderly have high rates of time preference due to the age-group effect, even if the last stage of life is excluded. The health cognition effect becomes important here. Since young adults are generally very healthy, we considered this effect to be largely irrelevant for them. It is only relevant for middle and old ages. Middle age is a period when health status and health-related issues become a concern, but the actual health status is stable and the perceived health is good. Accordingly, the rate of time preference exhibits a low value. It is often observed that health gradually deteriorates after individuals experience serious illnesses or are diagnosed with chronic diseases. This worsens the perception of health.[7] It is conceivable that lower health cognitions in the elderly may lead them to engage in ephemeral economic behavior, including consumption, given their limited remaining years of life. A higher rate of time preference corresponds to this situation.

Health issues, which are of great concern everywhere, and aging issues, which have become an important political challenge in developed countries, are closely related to how people face the present and the future, and therefore, various issues are concentrated in time preference (time discounting). The significance of treating these issues within the framework of a dynamic general equilibrium is sure to grow further over the coming years.

References

[1] Anchugina, N., Ryan, M., Slinko, A. (2017) Hyperbolic discounting of the far-distant future. *Economics Letters* 155, 140-143.

[2] Barro, R. J., Mankiw, N. G., Sala-i-Martin, X. (1995) Capital mobility in neoclassical models

7 If they are middle-aged, they may choose to live patiently (i.e., lower ρ) since they still have a long second half of their lives.

of growth. *American Economic Review* 85 (1), 103–115.

[3] Dalgaard, C.–J., Strulik, H. (2014) Optimal aging and death: Understanding the Preston curve. *Journal of the European Economic Association* 12 (3), 672–701.

[4] Das, M. (2003) Optimal growth with decreasing marginal impatience. *Journal of Economic Dynamics & Control* 27 (10), 1881–1898.

[5] Epstein, L., Hynes, J. A. (1983) The rate of time preference and dynamic economic analysis. *Journal of Political Economy* 91 (4), 611–635.

[6] Harrison, G. W., Lau, M. I., Williams, M. B. (2002) Estimating individual discount rates in Denmark: A field experiment. *American Economic Review* 92 (5), 1606–1617.

[7] Hirata, K. (2008) Age–specific heterogeneity in time preference rates. *Journal of Behavioral Economics and Finance* 1, 58–60 (in Japanese).

[8] Hosoya, K. (2023) Impact of infectious disease pandemics on individual lifetime consumption: An endogenous time preference approach. *Journal of Macroeconomics* 76, 103506.

[9] Jordà, Ò., Knoll, K., Kuvshinov, D., Schularick, M., Taylor, A. M. (2019) The rate of return on everything, 1870–2015. *Quarterly Journal of Economics* 134 (3), 1225–1298.

[10] Laibson, D. (1997) Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112 (2), 443–478.

[11] Meng, Q. (2006) Impatience and equilibrium indeterminacy. *Journal of Economic Dynamics & Control* 30 (12), 2671–2692.

[12] Mitnitski, A. B., Mogilner, A. J., MacKnight, C., Rockwood, K. (2002) The accumulation of deficits with age and possible invariants of aging. *TheScientificWorldJOURNAL* 2, 1816–1822.

[13] Nishimura, K., Shimomura, K. (2002) Indeterminacy in a dynamic small open economy. *Journal of Economic Dynamics & Control* 27 (2), 271–281.

[14] Obstfeld, M. (1990) Intertemporal dependence, impatience, and dynamics. *Journal of Monetary Economics* 26 (1), 45–75.

[15] Ortigueira, S., Santos, M. S. (1997) On the speed of convergence in endogenous growth models. *American Economic Review* 87 (3), 383–399.

[16] Raurich, X. (2003) Government spending, local indeterminacy and tax structure. *Economica* 70 (280), 639–653.

[17] Stokey, N. L., Rebelo, S. (1995) Growth effects of flat–rate taxes. *Journal of Political Economy* 103 (3), 519–550.

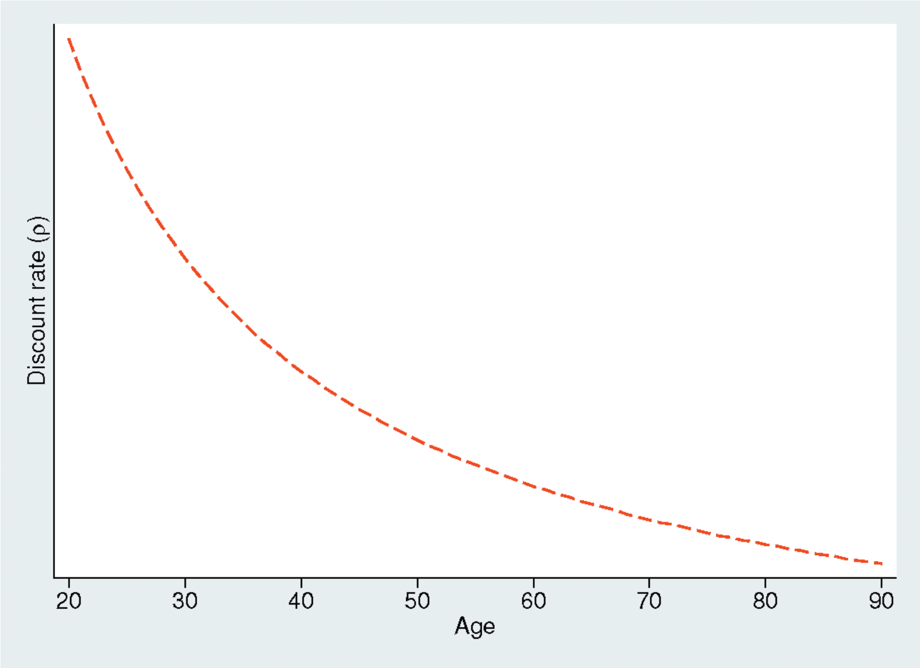
[18] Strulik, H., Trimborn, T. (2018) Hyperbolic discounting can be good for your health. *Journal of Economic Psychology* 69, 44–57.

[19] Strulik, H. (2021) Intertemporal choice with health–dependent discounting. *Mathematical Social Sciences* 111, 19–25.

[20] Uzawa, H. (1968) Time preference, the consumption function, and optimum asset holdings. In: Wolfe, J. N. (Ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*. Edinburgh University Press, Edinburgh, pp. 485–504.

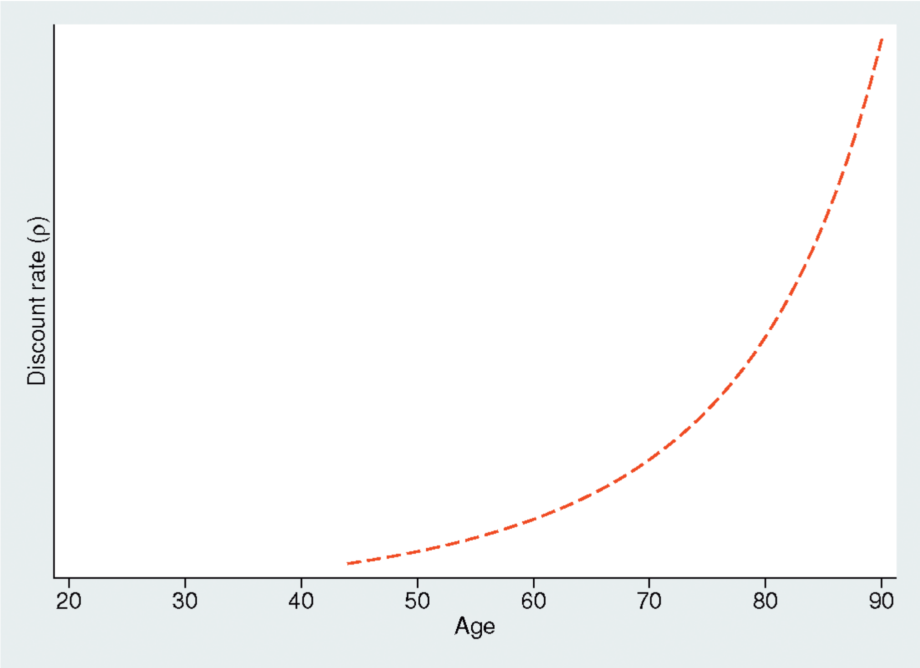
Appendix A. Hyperbolic discounting under age-group effects

Figure A1. Hyperbolic discounting under age-group effects.



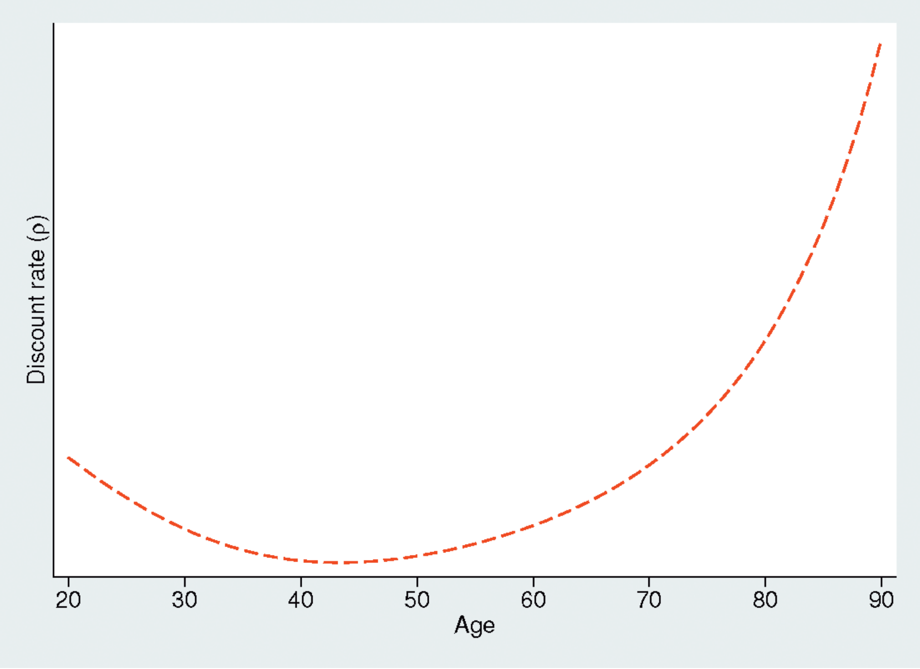
Appendix B. Increasing marginal impatience under health cognition effects

Figure B1. Increasing marginal impatience under health cognition effects.



Appendix C. Composite U-shaped time preference over a lifetime

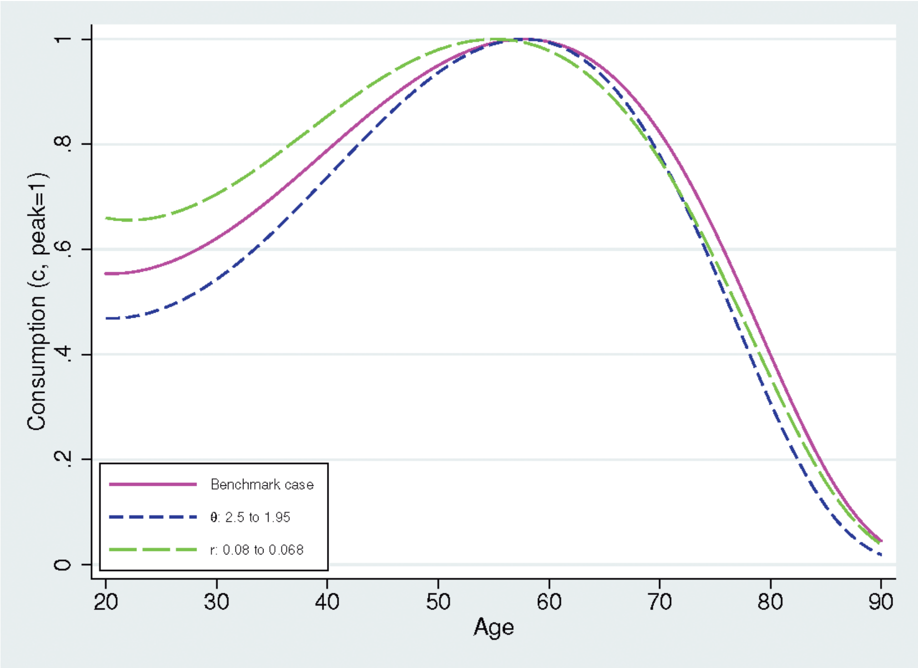
Figure C1. Composite U-shaped time preference over a lifetime.



Appendix D. Impact of changes in θ and r

The fixed parameters for estimating the lifetime consumption profiles were θ and r. We would like to show how the estimation results change when these parameters are revised within an appropriate range. The benchmark case set in Section 3 is used as the reference estimation here. First, the case where θ is changed from 2. 5 to 1. 95 is presented in Figure D1 along with the benchmark case. The case where r is changed from 0. 08 to 0. 068 is also included in Figure D1. See Section 3 for the employed parameters of the benchmark case.

Figure D1. Impact of changes in θ and r.



Note: $\bar{\rho}=-0.03$, $\eta_1=30$, $\eta_2=229.65$, $\eta_3=0.04$, $\eta_4=10.5$, $c(0)=4,000,000$, $r=(0.068,\ 0.08)$, and $\theta=(1.95,\ 2.5)$.